3rd International Mathematics Assessments for Schools (2013-2014)

Junior Division Round 2

Time: 120 minutes

Printed Name: ____________________ *Code*: ______________ *Score*: ____________

Instructions:

- Do not open the contest booklet until you are told to do so.
- Be sure that your name and code are written on the space provided above.
- Round 2 of IMAS is composed of three parts; the total score is 100 marks.
- Questions 1 to 5 are given as a multiple-choice test. Each question has five possible options marked as A, B, C, D and E. Only one of these options is correct. After making your choice, fill in the appropriate letter in the space provided. Each correct answer is worth 4 marks. There is no penalty for an incorrect answer.
- Questions 6 to 13 are a short answer test. Only Arabic numerals are accepted; using other written text will not be honored or credited. Some questions have more than one answer, as such all answers are required to be written down in the space provided to obtain full marks. Each correct answer is worth 5 marks. There is no penalty for incorrect answers.
- Questions 14 and 15 require a detailed solution or process in which 20 marks are to be awarded to a completely written solution. Partial marks may be given to an incomplete presentation. There is no penalty for an incorrect answer.
- Use of electronic computing devices is not allowed.
- Only pencil, blue or black ball-pens may be used to write your solution or answer.
- Diagrams are not drawn to scale. They are intended as aids only.
- After the contest the invigilator will collect the contest paper.

The following area is to be filled in by the judges;
the contestants are not supposed to mark anything here.

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Total Score	Signature
Score																	
Score																	

Junior Division Round 2

Questions 1 to 5, 4 marks each

1. When the digits 0, 1, 2, 5, 6, 8 and 9 are rotated 180°, they become 0, 1, 2, 5, 9, 8 and 6 respectively. What does 9105 become when the four-digit number is rotated 180°?

（A）6150 （B）6102 （C）5016 （D）2019 （E） 2016

Answer：________________

2. What is the value of the expression

$$(a-b)(a+b-c)+(b-c)(b+c-a)+(c-a)(c+a-b)$$

（A）0 （B）$a^2+b^2+c^2$ （C）$ab+bc+ca$

（D）$a^2+b^2+c^2-ab-bc-ca$ （E）$a^2+b^2+c^2+ab+bc+ca$

Answer：________________

3. We wish to dissect a square into n squares which need not be of the same size. For which of the following values of n is this impossible?

（A）5 （B）6 （C）7 （D）8 （E）9

Answer：________________

4. The total number of players on three badminton teams is 29. No two players on the same team play against each other, while every two players on different teams play each other exactly once. What is the maximum number of games played?

（A）265 （B）270 （C）276 （D）280 （E）282

Answer：________________

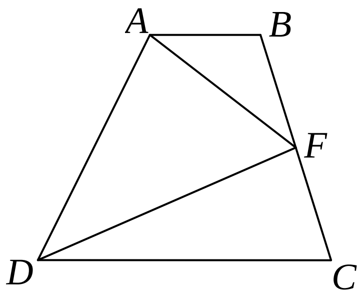
5. Two distinct quadratic polynomials $f(x)$ and $g(x)$ with leading coefficients equal to 1 satisfy $f(1)+f(3)+f(5)=g(1)+g(3)+g(5)$. Find all solutions of $f(x)=g(x)$.

（A）$x\le 0$ （B）$-2\le x\le 0$ （C）$0\le x\le 1$ （D）$2\le x\le 2$ （E）3

Answer：________________

Questions 6 to 13, 5 marks each

6. The diagram shows a quadrilateral *ABCD* with *AB* parallel to *DC*. *F* is the midpoint of *BC*. If the area of triangle *AFD* is 10 cm^2, what is the area, in cm^2, of *ABCD*?



Answer： ____________ cm^2

7. Lily has 2014 chocolates. She eats one on the first day. Each day after, she eats twice as many as the day before, until all the chocolates have been eaten. How many chocolates did she eat on the last day?

Answer： ____________ chocolates

8. Leon is putting 99 apples into boxes of two different sizes. A large box can hold 12 apples while a small box can hold 5 apples. All boxes must be full. How many boxes will he need if this number must be greater than 10?

Answer： ____________ boxes

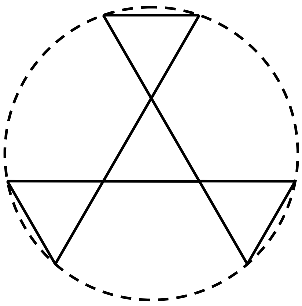
9. In a convex quadrilateral *ABCD*, *AB* = 3, *BC* = 5, *CD* = 6, *DA* = 10, and the length of the diagonal *AC* is a positive integer. How many different possible shapes can *ABCD* take?

Answer： ____________ shapes

10. Divide the ten positive integers from 1 to 10 into two groups so that when the product of the numbers in the first group is divided by the product of the numbers in the second group, the quotient is a positive integer. What is the minimum value of this quotient?

Answer：

11. A cardboard windmill with three blades is made from four equilateral triangles of side length 6 cm. Two triangles sharing a common vertex have the corresponding sides lying on the same straight line, as shown in the diagram. The area of the circle swept out by the blades of the windmill is x cm^2. What is the greatest integer less than or equal to x?



Answer：

12. Let the real numbers a_1, a_2, a_3, a_4 and a_5 be such that $a_{n+1} = |a_n| - |a_n - 1|$ for $1 \le n \le 4$. If $a_5 = \frac{1}{2}$ and $a_1 = \frac{p}{q}$, where p and q are relatively prime positive integers, what is the value of $p + q$?

Answer：

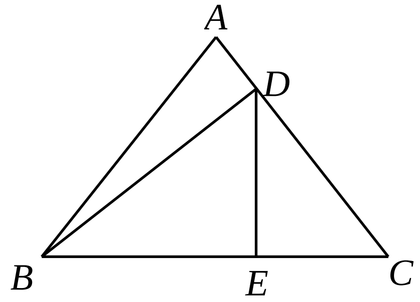
13. What is the minimum perimeter of a parallelogram which may be partitioned into 462 equilateral triangles of side length 1 cm?

Answer： cm

Questions 14 to 15, 20 marks each

Detailed solutions are needed for these two problems

14. In an acute triangle ABC, $AB = AC$. D is the foot of the perpendicular from B to CA, and E is the foot of the perpendicular from D to BC. If $BC = AB + AD$, prove that $BE = CD$.



15. A positive integer x with $n \geq 2$ digits is written down twice in a row and the $2n$-digit number so obtained is divisible by x^2. Prove that the first two digits of x are 1 and 4 in that order.